Spectral duality and Smilansky Hypothesis for the inner and the outer Neumann Laplacean in $R_3 = \Omega_{int} \cup \Omega_{out}$.

G. Martin[1], B. Pavlov[1,2]

[1] New Zealand Institute of Advanced Study, Massey University, New Zealand.

[2] V.A. Fock Institute for Physics of St.-Petersburg University, Petrodvorets, Russia.

Let Γ be a zero-thin smooth shell separating R_3 into two domains $\Omega_{int} \cup \Omega_{out}$. Consider the Neumann Laplacean L^N_{int}, L^N_{out}. The spectrum of L^N_{int} is discrete, and one of L^N_{out} is purely continuous. The eigenfunctions of L^N_{out} are scattered waves $\psi(x,\nu,\lambda), \lambda = k^2 \geq 0$, which satisfy the Helmholtz equation, $-\Delta\psi = p^2\psi$, the Neumann boundary condition on the shell: $\frac{\partial\psi}{\partial n}\Big|_\Gamma = 0$, and the asymptotic boundary condition at infinity

$$\psi(x,\nu,k) \approx e^{ip\langle x,\nu\rangle} - 2\pi^2 a(\omega,\nu,k)\frac{e^{-ip|x|}}{|x|}, \text{ when } x \to \omega\ \infty.$$

The coefficient $a(\omega,\nu,p)$ is the scattering amplitude. The integral operator

$$S^N : u \longrightarrow u - i\pi p \int_{\Sigma_2} a(\omega,\nu)u(\nu)d\Sigma(\nu) = (I + T^N)u$$

is unitary in $L_2(\Sigma_2)$ for each p, $p > 0$. It is called the Scattering Matrix, and T^N is called Neumann T-matrix. Similarly the Dirichlet Scattering matrix and T-matrix are defined.

Uzy Smilansky formulated in 1995 a hypothesis concerning spectral duality of the inner and outer Dirichlet Laplacean L^D_{int}, L^D_{out}: the vector-zeros $(p_0,\ e_0)$, $T^D(p_0)e_0 = 0$, of the Dirichlet T-matrix are the eigenvalues of the corresponding inner Dirichlet problem and, vice versa, the eigenvalues of the inner Dirichlet problem coincide with the vector-zeros of the Dirichlet T-matrix of the corresponding outer problem. The hypothesis was checked numerically [1], and soon proved analytically for 2-d Dirichlet Laplacian [2].

We suggest a proof of the spectral duality for the inner and the outer Neumann Laplacean in $L_2(R_3)$. Our proof is extendedable to Neumann Laplacean in all finite-dimensional Euclidean spaces.

References

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[2] J.-P.Eckman, C.-A. Pillet *Spectral duality for planar billiards* In : Commun. mathematical Physics, **170**, 2 (1995),pp 283 - 313.