A correction in the statement of my theorem on the efficient smooth cover and smooth stratification of an algebraic variety

Alexander L. Chistov

St. Petersburg Institute for Informatics and Automation of the Academy of Sciences of Russia
14th line 39, St. Petersburg 199178, Russia,
e-mail: labta@iias.spb.su or sliss@iias.spb.su

2004

Abstract

We correct the statement of our theorem about the construction of the efficient smooth cover and smooth stratification of an algebraic variety in zero–characteristic. Namely, this correction is related to the estimations of the lengths of the coefficients from the ground field of the polynomials giving the strata. The proof of this theorem, the bounds for degrees of the strata and all the consequences of the theorem are without changes.

In [1] Theorem 2 we prove the existence of an efficient smooth cover and smooth stratification an algebraic variety in zero–characteristic. Assertion (vi) of Theorem 2 from [1] must be corrected as follows:

(vi) *For all* $\alpha \in A$, $1 \le j \le s(\alpha)$ *the lengths of coefficients from* k *of polynomials* $h_{\alpha,j}$ *are bounded from above by a polynomial in* $n^{2^{s(\alpha)^C}} d^{ns(\alpha)}$, d_1, d_2, M, M_1, m *for an absolute constant* $0 < C \in \mathbb{R}$. *Further, in the case of smooth stratification the lengths of coefficients from* k *of all polynomials of the family* f *are bounded from above by a polynomial in* $2^{2^{n^C}} d^{n^2}$, d_1, d_2, M, M_1, m *for an absolute constant* $0 < C \in \mathbb{R}$.

Thus, one needs to replace d^n by $d^{ns(\alpha)}$ for the case of smooth cover (respectively by d^{n^2} for the case of the smooth stratification) in (vi) Theorem 2 [1]. All the other assertions of this theorem and its proof are without changes. In particular, the degrees of all the strata are bounded from above by $2^{2^{n^C}} d^n$. The proof is the same since actually we did not estimate the lengths of coefficients from k of these polynomials explicitly. The bound from (vi) is obtained straightforwardly by the recursive application of Theorem 1 [1], see the proof Theorem 2 [1].

This correction implies the same correction in the same assertion (vi) of Theorem 4 from [2]. The working time of the algorithm from Theorem 4 [2] for constructing the smooth cover (respectively the smooth stratification) is polynomial in the size of the output and $2^{2^{n^C}} d^n$, d_1, d_2, M, M_1, m (respectively is polynomial in $2^{2^{n^C}} d^{n^2}$, d_1, d_2, M, M_1, m), where $0 < C \in \mathbb{R}$ is an absolute constant. So in the case of the smooth cover we need to add here "in the size of the output" which might be polynomial also d^{n^2} by (vi).

Notice that the proof of Theorem 3 [1] about computing the dimension of a real algebraic variety is the same. It uses only the fact that the degrees of the strata from Theorem 2 [1] are bounded from above by $2^{2^{n^C}} d^n$. But now one can not use immediately Theorem 4 [2] to deduce Theorem 3 [1]. So I know only one proof of Theorem 3 [1] using the original construction from [1].

References

[1] **Chistov A. L.:** *"Efficient Smooth Stratification of an Algebraic Variety in Zero–Characteristic and its Applications*, Zap. Nauchn. Semin. St-Petersburg. Otdel. Mat. Inst. Steklov (POMI) v. 266 (2000) p.254–311 (in Russian)

[2] **Chistov A. L.:** *"Polynomial–time algorithms in zero–characteristic for a new model of representation of algebraic varieties"*, Preprint of the St.Petersburg Mathematical Society (2004) #10, http://www.MathSoc.spb.ru/preprints/