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On Homogenization for Non-Self-Adjoint Periodic Elliptic Operators on an Infinite Cylinder*

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Abstract. We consider an operator $\mathscr{A}^\varepsilon$ on $L_2(\mathbb{R}^{d_1}\times\mathbb{T}^{d_2})$ (d_1 is positive, while d_2 can be zero) given by $\mathscr{A}^\varepsilon=-\operatorname{div}A(\varepsilon^{-1}x_1,x_2)\,\nabla$, where A is periodic in the first variable and smooth in a sense in the second. We present approximations for $(\mathscr{A}^\varepsilon-\mu)^{-1}$ and $\nabla(\mathscr{A}^\varepsilon-\mu)^{-1}$ (with appropriate μ) in the operator norm when ε is small. We also provide estimates for the rates of approximation that are sharp with respect to the order.

Key words: homogenization, operator error estimates, periodic differential operators, effective operator, corrector.

1. Problem setting. The present paper is concerned with the study of homogenization problems for operators whose coefficients are periodic only in some directions. Let $d_1>0$ be the number of "periodic" directions and $d_2\geqslant 0$ be the number of "nonperiodic" directions; we denote their sum by d. We set $\Xi=\mathbb{R}^{d_1}\times\Omega_2$, where $\Omega_2=\mathbb{T}^{d_2}$ is the flat torus $(\mathbb{R}/\mathbb{Z})^{d_2}$. Let Λ be a lattice in $\mathbb{R}^{d_1}$ with basic cell Ω_1. We use Λ^* and Ω_1^* to denote the dual lattice and the Brillouin zone, respectively. It is convenient for us here to view Λ as acting on Ξ, so that $\Omega=\Omega_1\times\Omega_2$ is a fundamental domain for Λ.

We set $\mathscr{D}=-i\nabla_x$. We also introduce the notation $\mathscr{D}_1=\binom{-i\nabla_{x_1}}{0}$ and $\mathscr{D}_2=\binom{0}{-i\nabla_{x_2}}$. Suppose that A is a periodic (with respect to the lattice) matrix-valued function in $\operatorname{Lip}(\Omega_2;L_\infty(\Omega_1))^{d\times d}$. Assume also that the real part of A is uniformly positive definite. We consider the strictly m-sectorial operator $\mathscr{A}^\varepsilon_\mu$ on $L_2(\Xi)$ associated with the form

$$\mathscr{a}^\varepsilon_\mu[u]=(A^\varepsilon\mathscr{D}u,\mathscr{D}u)_{L_2(\Xi)}-\mu(u,u)_{L_2(\Xi)},\qquad \mu\in\mathbb{C},\ \varepsilon>0,\tag{1}$$

defined for all u in the Sobolev space $H^1(\Xi)$. Here and henceforward, if f is a function on Ξ, then the symbol f^ε stands for the mapping $f^\varepsilon(x)=f(\varepsilon^{-1}x_1,x_2)$.

The coefficients of $\mathscr{A}^\varepsilon_\mu$ rapidly oscillate along the periodic directions when ε becomes small. Our goal is to study the behavior of $(\mathscr{A}^\varepsilon_\mu)^{-1}$ as ε goes to 0. We show that $(\mathscr{A}^\varepsilon_\mu)^{-1}$ and $\mathscr{D}_2(\mathscr{A}^\varepsilon_\mu)^{-1}$ converge in the operator norm and prove sharp-order estimates for the rates of convergence. Moreover, we find an approximation for $\mathscr{D}_1(\mathscr{A}^\varepsilon_\mu)^{-1}$ and obtain a more accurate approximation for $(\mathscr{A}^\varepsilon_\mu)^{-1}$ with error of order ε^2.

Such problems have already been studied, e.g., in [3] and [5]. However, the results of these papers are related to operators subject to fairly strong restrictions (in particular, the matrix A must be Hermitian and have block-diagonal structure), which cannot be relaxed within the framework of approaches used there. In this paper we present a relatively simple approach that does not have this flaw. At the same time, this approach makes it possible to not only generalize the previous results but also obtain other, more subtle, results. Namely, our method gives, in addition to the convergence results, a more accurate approximation for non-self-adjoint operators in the operator norm on L_2, involving a corrector. Earlier, such results were obtained only by a spectral approach and only for purely periodic operators on the entire space (see [1], [2], and [6]).

We dwelt on a problem with periodic boundary conditions and operators not containing lower-order terms only to simplify the statements of results. A more general case, where the operator has lower-order terms with multiplier coefficients, is investigated in [4].

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2. Main results. For our purposes, we will need an effective operator and two different correctors. Let N be the $\mathbb{C}^{1\times d}$-valued periodic solution of the following auxiliary problem on the basic cell:

$$\mathscr{D}_1^* A(\,\cdot\,, x_2)(\mathscr{D}_1 N(\,\cdot\,, x_2) + I) = 0, \qquad \int_{\Omega_1} N(\,\cdot\,, x_2)\, dx_1 = 0.$$

Then the effective operator $\mathscr{A}_\mu^0$ is the strictly m-sectorial operator given by

$$\mathscr{A}_\mu^0 = \mathscr{D}^* A^0 \mathscr{D} - \mu, \tag{2}$$

where

$$A^0(x_2) = |\Omega_1|^{-1} \int_{\Omega_1} A(y_1, x_2)(\mathscr{D}_1 N(y_1, x_2) + I)\, dy_1.$$

Note that the function A^0 is Lipschitz continuous and its real part is uniformly positive definite, so that the domain of the effective operator coincides with the Sobolev space $H^2(\Xi)$.

It is easy to see that the spectra of $\mathscr{A}_0^0$ and $\mathscr{A}_0^\varepsilon$ are contained in a sector $S \subset \{z \in \mathbb{C}\colon\ \operatorname{Re} z > 0\}$. Hence both $\mathscr{A}_\mu^\varepsilon$ and $\mathscr{A}_\mu^0$ have bounded inverses for any μ outside the sector. Our first result concerns the convergence of $(\mathscr{A}_\mu^\varepsilon)^{-1}$ and $\mathscr{D}_2(\mathscr{A}_\mu^\varepsilon)^{-1}$.

Theorem 1. *Let $\mu \notin S$. Then, for all $\varepsilon \in (0, 1]$,*

$$\|(\mathscr{A}_\mu^\varepsilon)^{-1} - (\mathscr{A}_\mu^0)^{-1}\|_{\mathbf{B}(L_2(\Xi))} \leqslant C_1 \varepsilon, \tag{3}$$

$$\|\mathscr{D}_2(\mathscr{A}_\mu^\varepsilon)^{-1} - \mathscr{D}_2(\mathscr{A}_\mu^0)^{-1}\|_{\mathbf{B}(L_2(\Xi))^d} \leqslant C_2 \varepsilon. \tag{4}$$

The estimates are sharp with respect to the order, and the constants C_1 and C_2 depend only on Λ, μ, $\|A\|_{L_\infty}$, $\|\mathscr{D}_2 A\|_{L_\infty}$, and $\|(\operatorname{Re} A)^{-1}\|_{L_\infty}$.

We now turn to the definition of the correctors. The first one, $\mathscr{K}_\mu^\varepsilon$, plays the role of the classical corrector in homogenization theory and differs from the latter in that it involves a smoothing operator $\mathscr{P}^\varepsilon$:

$$\mathscr{K}_\mu^\varepsilon = N^\varepsilon \mathscr{D} (\mathscr{A}_\mu^0)^{-1} \mathscr{P}^\varepsilon. \tag{5}$$

As $\mathscr{P}^\varepsilon$ we take the pseudodifferential operator with symbol $\chi_{\varepsilon^{-1}\Omega_1^*}$ ($\chi_{\varepsilon^{-1}\Omega_1^*}$ being the characteristic function of the set $\varepsilon^{-1}\Omega_1^*$):

$$\mathscr{P}^\varepsilon = \mathscr{F}^* \chi_{\varepsilon^{-1}\Omega_1^*} \mathscr{F},$$

where $\mathscr{F}$ is the partial Fourier transform in the variable x_1. In certain cases, $\mathscr{K}_\mu^\varepsilon$ can be replaced by the classical corrector (see [4]).

Theorem 2. *Let $\mu \notin S$. Then, for any $\varepsilon \in (0, 1]$,*

$$\|\mathscr{D}_1(\mathscr{A}_\mu^\varepsilon)^{-1} - \mathscr{D}_1(\mathscr{A}_\mu^0)^{-1} - \varepsilon \mathscr{D}_1 \mathscr{K}_\mu^\varepsilon\|_{\mathbf{B}(L_2(\Xi))^d} \leqslant C_3 \varepsilon. \tag{6}$$

The estimate is sharp with respect to the order, and the constant C_3 depends only on Λ, μ, $\|A\|_{L_\infty}$, $\|\mathscr{D}_2 A\|_{L_\infty}$, and $\|(\operatorname{Re} A)^{-1}\|_{L_\infty}$.

Notice that, because of the rapidly oscillating function N^ε, the norm of the corrector in $\mathbf{B}(L_2(\Xi), H^1(\Xi))$ is of order ε^{-1}. It follows that we cannot generally eliminate $\mathscr{K}_\mu^\varepsilon$ from (6).

The second corrector, $\mathscr{C}_\mu^\varepsilon$, appeared for the first time in [1] and has a rather complicated structure. Let $k \in \mathbb{R}^{d_1}$, and let $\mathscr{k} = \binom{k}{0}$ be the corresponding element of $\mathbb{R}^d$. We introduce two families of operators:

$$\mathscr{A}_\mu^0(k) = (\mathscr{k} + \mathscr{D}_2)^* A^0 (\mathscr{k} + \mathscr{D}_2) - \mu,$$

$$\mathscr{K}_\mu(k; x_1) = N(x_1, \,\cdot\,)\,(\mathscr{k} + \mathscr{D}_2)(\mathscr{A}_\mu^0(k))^{-1}.$$

Let us denote the adjoint of $\mathscr{A}_\mu^\varepsilon$ by $(\mathscr{A}_\mu^\varepsilon)^+$. We construct the effective operator, the corrector, and the families for $(\mathscr{A}_\mu^\varepsilon)^+$ just as we did for $\mathscr{A}_\mu^\varepsilon$ (they will be marked with "+" as well). Let $\mathscr{L}_\mu$ be the pseudodifferential operator (in the variable x_1) with symbol

$$k \mapsto \mathscr{L}_\mu(k) = |\Omega_1|^{-1} \int_{\Omega_1} (\mathscr{K}_\mu(k; y_1)^+)^* (\mathscr{k} + \mathscr{D}_2)^* A(y_1, \,\cdot\,)\,((\mathscr{k} + \mathscr{D}_2)(\mathscr{A}_\mu^0(k))^{-1} + \mathscr{D}_{y_1} \mathscr{K}_\mu(k; y_1))\, dy_1.$$

Then the corrector $\mathscr{C}^\varepsilon_\mu$ is defined by

$$\mathscr{C}^\varepsilon_\mu = (\mathscr{K}^\varepsilon_\mu - \mathscr{L}_\mu)\mathscr{P}^\varepsilon + \mathscr{P}^\varepsilon((\mathscr{K}^\varepsilon_\mu)^+ - (\mathscr{L}_\mu)^+)^*. \tag{7}$$

We may as well express the operator $\mathscr{L}_\mu$ in a different form, namely,

$$\mathscr{L}_\mu = (\mathscr{D}((\mathscr{A}^0_\mu)^+)^{-1})^* \mathscr{M}(\mathscr{D}(\mathscr{A}^0_\mu)^{-1}),$$

where $\mathscr{M}$ is the pseudodifferential operator (in the variable x_1) with symbol

$$k \mapsto \mathscr{M}(k) = |\Omega_1|^{-1} \int_{\Omega_1} (N(y_1, \cdot)^+)^* (\not{k} + \mathscr{D}_2)^* A(y_1, \cdot)\,(I + (\mathscr{D}_1 N)(y_1, \cdot))\,dy_1.$$

(It is easy to see that $\mathscr{M}$ is in fact a differential operator.) A similar definition was given in [1].

Theorem 3. *Let $\mu \notin S$. Then, for any $\varepsilon \in (0, 1]$,*

$$\|(\mathscr{A}^\varepsilon_\mu)^{-1} - (\mathscr{A}^0_\mu)^{-1} - \varepsilon \mathscr{C}^\varepsilon_\mu\|_{\mathbf{B}(L_2(\Xi))} \leqslant C_4 \varepsilon^2. \tag{8}$$

The estimate is sharp with respect to the order, and the constant C_4 depends only on Λ, μ, $\|A\|_{L_\infty}$, $\|\mathscr{D}_2 A\|_{L_\infty}$, and $\|(\operatorname{Re} A)^{-1}\|_{L_\infty}$.

We remark that the smoothing $\mathscr{P}^\varepsilon$ can always be dropped from (7); Theorem 3 will remain true (see [4]).

3. The method. First, we note that it suffices to prove the theorems for only one $\mu \notin S$; the result for the remaining values of μ will then follow by using an appropriate identity for the resolvents. It is convenient to choose μ with $\operatorname{Re}\mu < 0$, since, in this case, the operators are coercive.

As is customary in problems of this kind, we use a scaling transformation and the Gelfand transform with respect to the first variable and reduce the problem to one on the fundamental domain Ω of Λ.

Let $\tau = (k, \varepsilon) \in \Omega^*_1 \times (0, 1]$. We introduce the notation $\mathscr{D}_1(\tau) = \mathscr{D}_1 + \not{k}$, $\mathscr{D}_2(\tau) = \varepsilon \mathscr{D}_2$, and $\mathscr{D}(\tau) = \mathscr{D}_1(\tau) + \mathscr{D}_2(\tau)$ and define the form $a_\mu(\tau)$ by

$$a_\mu(\tau)[u] = (A\mathscr{D}(\tau)u, \mathscr{D}(\tau)u)_{L_2(\Omega)} - \varepsilon^2 \mu (u, u)_{L_2(\Omega)},$$

where u is any function in the Sobolev space $\widetilde{H}^1(\Omega)$ of periodic functions belonging to $H^1(\Omega)$. This form generates an m-sectorial operator on $L_2(\Omega)$. We denote it by $\mathscr{A}_\mu(\tau)$. The relationship between $\mathscr{A}^\varepsilon_\mu$ and $\mathscr{A}_\mu(\tau)$ is rather simple: it can be proved that $\varepsilon^2 \mathscr{A}^\varepsilon_\mu$ is unitarily equivalent to a decomposable operator on $\int^\oplus_{\Omega^*_1} L_2(\Omega)\,dk$, whose fibers turn out to be $\mathscr{A}_\mu(\tau)$; that is,

$$\varepsilon^2 \mathscr{A}^\varepsilon_\mu \simeq \int^\oplus_{\Omega^*_1} \mathscr{A}_\mu(\tau)\,dk.$$

The fibers $\mathscr{A}^0_\mu(\tau)$ of the decomposable operator that is unitarily equivalent to $\varepsilon^2 \mathscr{A}^0_\mu$ are defined likewise. Next, let

$$\mathscr{K}_\mu(\tau) = N\mathscr{D}(\tau)(\mathscr{A}^0_\mu(\tau))^{-1}\mathscr{P}_1,$$

where $\mathscr{P}_1$ is averaging over Ω_1. Then

$$\varepsilon^{-1} \mathscr{K}^\varepsilon_\mu \simeq \int^\oplus_{\Omega^*_1} \mathscr{K}_\mu(\tau)\,dk.$$

Finally, if

$$\mathscr{L}_\mu(\tau) = (\mathscr{K}_\mu(\tau)^+)^* (\mathscr{D}(\tau) - \mathscr{D}_1)^* A((\mathscr{D}(\tau) - \mathscr{D}_1)(\mathscr{A}^0_\mu(\tau))^{-1} + \mathscr{D}_1 \mathscr{K}_\mu(\tau)),$$

then

$$\varepsilon^{-1} \mathscr{C}^\varepsilon_\mu \simeq \int^\oplus_{\Omega^*_1} \mathscr{C}_\mu(\tau)\,dk,$$

where $\mathscr{C}_\mu(\tau)$ is given by

$$\mathscr{C}_\mu(\tau) = (\mathscr{K}_\mu(\tau) - \mathscr{L}_\mu(\tau))\mathscr{P}_1 + \mathscr{P}_1(\mathscr{K}_\mu(\tau)^+ - \mathscr{L}_\mu(\tau)^+)^*.$$

Now we can rewrite Theorems 1–3 in terms of the corresponding fibers.

Theorem 1*. *For any* $\tau \in \Omega_1^* \times (0,1]$,

$$\|(\mathscr{A}_\mu(\tau))^{-1} - (\mathscr{A}_\mu^0(\tau))^{-1}\|_{\mathbf{B}(L_2(\Omega))} \leqslant C_1|\tau|^{-1}, \tag{9}$$

$$\|\mathscr{D}_2(\tau)(\mathscr{A}_\mu(\tau))^{-1} - \mathscr{D}_2(\tau)(\mathscr{A}_\mu^0(\tau))^{-1}\|_{\mathbf{B}(L_2(\Omega))^d} \leqslant C_2. \tag{10}$$

Theorem 2*. *For any* $\tau \in \Omega_1^* \times (0,1]$,

$$\|\mathscr{D}_1(\tau)(\mathscr{A}_\mu(\tau))^{-1} - \mathscr{D}_1(\tau)(\mathscr{A}_\mu^0(\tau))^{-1} - \mathscr{D}_1(\tau)\mathscr{K}_\mu(\tau)\|_{\mathbf{B}(L_2(\Omega))^d} \leqslant C_3. \tag{11}$$

Theorem 3*. *For any* $\tau \in \Omega_1^* \times (0,1]$,

$$\|(\mathscr{A}_\mu(\tau))^{-1} - (\mathscr{A}_\mu^0(\tau))^{-1} - \mathscr{C}_\mu(\tau)\|_{\mathbf{B}(L_2(\Omega))} \leqslant C_4. \tag{12}$$

Let us briefly sketch the key ideas of the proof. The starting point is the relation

$$\begin{aligned}(\mathscr{A}_\mu(\tau))^{-1}\mathscr{P}_1 &- (\mathscr{A}_\mu^0(\tau))^{-1}\mathscr{P}_1 - \mathscr{K}_\mu(\tau)\\ &= -(\mathscr{A}_\mu(\tau))^{-1}\mathscr{P}_1^\perp(\mathscr{D}(\tau) - \mathscr{D}_1)^* A((\mathscr{D}(\tau) - \mathscr{D}_1)(\mathscr{A}_\mu^0(\tau))^{-1}\mathscr{P}_1 + \mathscr{D}_1\mathscr{K}_\mu(\tau))\\ &\quad + \varepsilon^2\mu(\mathscr{A}_\mu(\tau))^{-1}\mathscr{K}_\mu(\tau) - (\mathscr{D}(\tau)(\mathscr{A}_\mu(\tau)^+)^{-1})^* A(\mathscr{D}(\tau) - \mathscr{D}_1)\mathscr{K}_\mu(\tau),\end{aligned} \tag{13}$$

which is a consequence of the resolvent identity and the definitions of A^0 and N. We begin with Theorem 2*:

$$\begin{aligned}\|\mathscr{D}_1(\tau)(\mathscr{A}_\mu(\tau))^{-1} &- \mathscr{D}_1(\tau)(\mathscr{A}_\mu^0(\tau))^{-1} - \mathscr{D}_1(\tau)\mathscr{K}_\mu(\tau)\|_{\mathbf{B}(L_2(\Omega))^d}\\ &\leqslant \|\mathscr{D}_1(\tau)(\mathscr{A}_\mu(\tau))^{-1}\mathscr{P}_1 - \mathscr{D}_1(\tau)(\mathscr{A}_\mu^0(\tau))^{-1}\mathscr{P}_1 - \mathscr{D}_1(\tau)\mathscr{K}_\mu(\tau)\|_{\mathbf{B}(L_2(\Omega))^d}\\ &\quad + \|\mathscr{D}_1(\tau)(\mathscr{A}_\mu(\tau))^{-1}\mathscr{P}_1^\perp\|_{\mathbf{B}(L_2(\Omega))^d} + \|\mathscr{D}_1(\tau)(\mathscr{A}_\mu^0(\tau))^{-1}\mathscr{P}_1^\perp\|_{\mathbf{B}(L_2(\Omega))^d}.\end{aligned}$$

Recall that the coefficients A and A^0 are Lipschitz continuous in the second variable and that the parameter μ was chosen in such a way that $\mathscr{A}_\mu(\tau)$ and $\mathscr{A}_\mu^0(\tau)$ are coercive; hence the estimates on $(\mathscr{A}_\mu(\tau))^{-1}$ and $(\mathscr{A}_\mu^0(\tau))^{-1}$, as well as on the compositions of these operators with $\mathscr{D}_1(\tau)$ and $\mathscr{D}_2(\tau)$. This, together with the fact that N is a Sobolev multiplier, gives the necessary estimates for $\mathscr{K}_\mu(\tau)$ and the compositions of $\mathscr{K}_\mu(\tau)$ with $\mathscr{D}_1(\tau)$ and $\mathscr{D}_2(\tau)$. Finally, all these estimates obviously carry over to the adjoint of $\mathscr{A}_\mu(\tau)$ and the related operators. Using identity (13) and applying the results mentioned above to the terms on the right-hand side, we obtain (11).

The estimates (9) and (10) of Theorem 1* are proved in the same way; we need only observe that $\mathscr{K}_\mu(\tau)$ and $\mathscr{D}_2(\tau)\mathscr{K}_\mu(\tau)$ can now be absorbed into the error terms.

As for (12), we note that, according to the definition of $\mathscr{C}_\mu(\tau)$, the operator under the norm sign can be written as the sum of the three terms

$$\begin{aligned}\mathscr{T}_1 &= (\mathscr{P}_1^\perp(\mathscr{A}_\mu(\tau)^+)^{-1} - \mathscr{P}_1^\perp(\mathscr{A}_\mu^0(\tau)^+)^{-1} - \mathscr{P}_1^\perp\mathscr{K}_\mu(\tau)^+)^*,\\ \mathscr{T}_2 &= (\mathscr{A}_\mu(\tau))^{-1}\mathscr{P}_1 - (\mathscr{A}_\mu^0(\tau))^{-1}\mathscr{P}_1 - \mathscr{K}_\mu(\tau),\\ \mathscr{T}_3 &= \mathscr{L}_\mu(\tau)\mathscr{P}_1 + (\mathscr{L}_\mu(\tau)^+\mathscr{P}_1)^*.\end{aligned}$$

The first one is estimated by employing the Poincaré inequality and an analogue of Theorem 2* for $\mathscr{A}_\mu^0(\tau)^+$. Next, we use identity (13) for $\mathscr{T}_2$ and then combine the terms in the sum of $\mathscr{T}_2$ and $\mathscr{T}_3$ so as to single out the operators

$$\begin{gathered}\mathscr{P}_1^\perp(\mathscr{A}_\mu(\tau)^+)^{-1} - \mathscr{P}_1^\perp(\mathscr{A}_\mu^0(\tau)^+)^{-1} - \mathscr{P}_1^\perp\mathscr{K}_\mu(\tau)^+,\\ \mathscr{D}(\tau)(\mathscr{A}_\mu(\tau)^+)^{-1} - \mathscr{D}(\tau)(\mathscr{A}_\mu^0(\tau)^+)^{-1} - \mathscr{D}(\tau)\mathscr{K}_\mu(\tau)^+.\end{gathered}$$

We apply analogues of Theorems 1* and 2* again to each term containing one of these operators. Thanks to $\mathscr{L}_\mu(\tau)$ and $\mathscr{L}_\mu(\tau)^+$ in the corrector fiber $\mathscr{C}_\mu(\tau)$, the remaining terms present no difficulties and can be handled by the same arguments as in the proofs of Theorems 1* and 2*.

References

[1] M. Sh. Birman and T. A. Suslina, Algebra i Analiz, **17**:6 (2005), 1–104; English transl.: St. Petersburg Math. J., **17**:6 (2006), 897–973.
[2] S. E. Pastukhova, Mat. Zametki, **94**:1 (2013), 130–150; English transl.: Math. Notes, **94**:1 (2013), 127–145.
[3] N. N. Senik, Algebra i Analiz, **25**:4 (2013), 182–259; English transl.: St. Petersburg Math. J., **25**:4 (2014), 647–697.
[4] N. N. Senik, `http://arxiv.org/abs/1508.04963`.
[5] T. A. Suslina, Algebra i Analiz, **16**:1 (2004), 269–292; English transl.: St. Petersburg Math. J., **16**:1 (2005), 237–257.
[6] T. A. Suslina, Algebra i Analiz, **26**:4 (2014), 195–263; English transl.: St. Petersburg Math. J., **26**:4 (2015), 643–693.

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